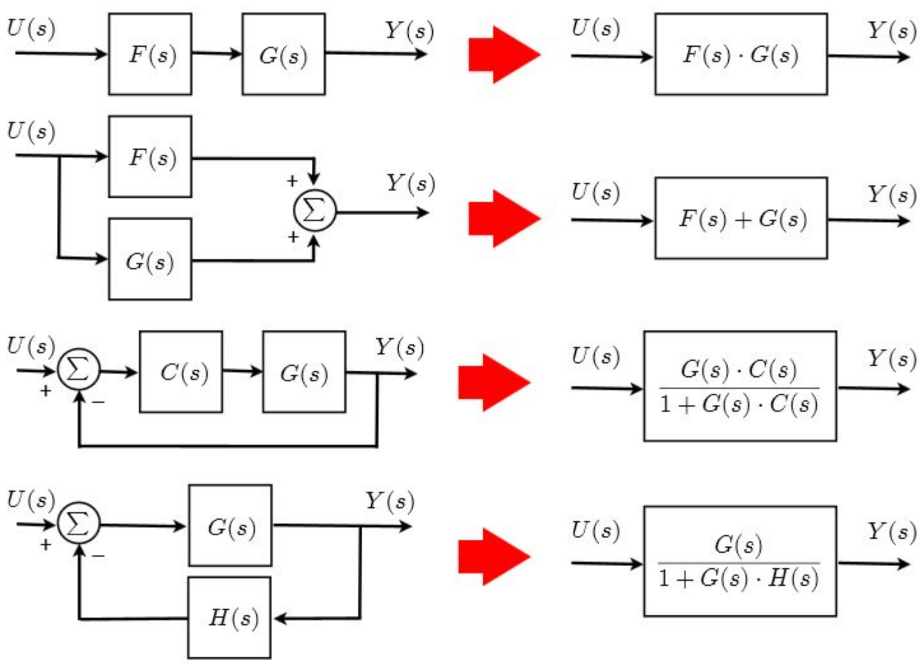
Misc

Proper Function
Degree of numerator does not exceed that of denominator

Strictly Proper Function
Degree of denominator exceeds degree of numerator

Block Diagrams

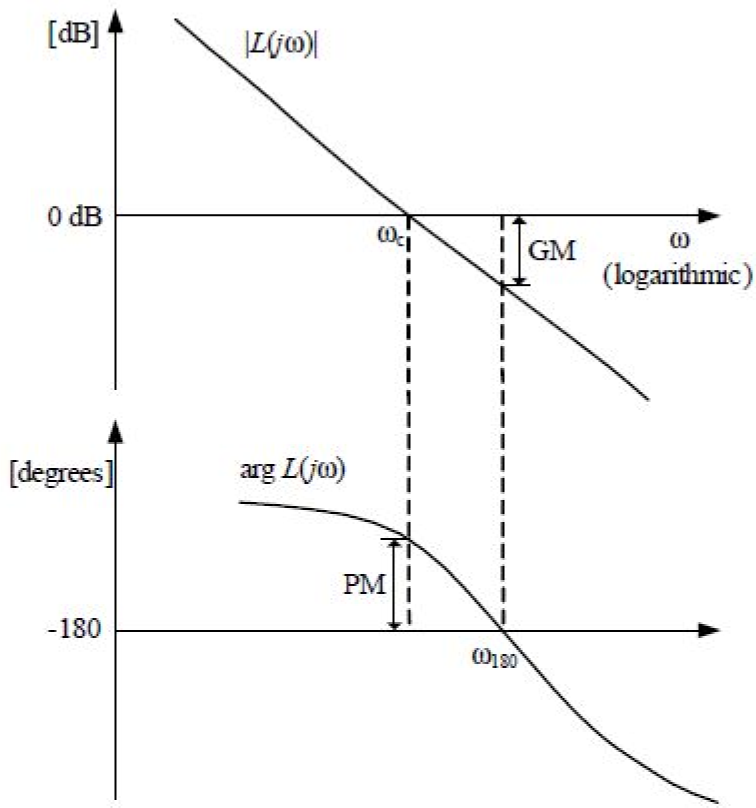


Linearization

$$
\begin{aligned}
0 &= \nabla h|_{\mathbf{x}_0} \cdot (\mathrm{x} - \mathrm{x}_0) \\
&= \frac{\partial h}{\partial y}\bigg|_{\mathbf{x}_0} \Delta y + \frac{\partial h}{\partial \dot{y}}\bigg|_{\mathbf{x}_0} \Delta \dot{y} + \frac{\partial h}{\partial \ddot{y}}\bigg|_{\mathbf{x}_0} \Delta \ddot{y} + \frac{\partial h}{\partial r}\bigg|_{\mathbf{x}_0} \Delta r + \frac{\partial h}{\partial \dot{r}}\bigg|_{\mathbf{x}_0} \Delta \dot{r}.
\end{aligned}
$$

$\Delta y = (y - yeq)$, *etc.......*

Gain and Phase Margin



$$PM = 180° + \angle G(j\omega_{gc}), \qquad GM = 1/|G(j\omega_{pc})|$$

Second Order Dynamics

$x^2 + 2\zeta\omega_n x + \omega_n{}^2 = 0$

Damping

$\omega_n = Natural\ Frequency$

$\zeta = Damping\ Ratio$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

Settling Time

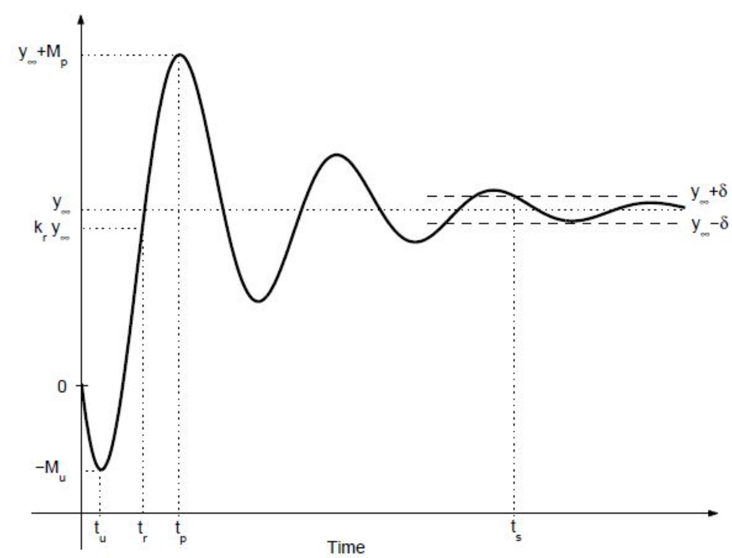
$t_s \approx \frac{4}{(\psi\omega_n)}$

Peak Time

$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

Overshoot

$\%OS = 100e^{-(\zeta\pi/\sqrt{1-\zeta^2}}$



Laplace Table

Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p,\ p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

Other Mathematical Tables

S.No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{(x-a)} + \frac{B}{(x-b)}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-b)}$
5.	$\frac{px^2+qx+r}{(x-a)^3(x-b)}$	$\frac{A}{(x-a)} + \frac{B}{(x-a)^2} + \frac{C}{(x-a)^3} + \frac{D}{(x-b)}$
6.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{(x-a)} + \frac{Bx+C}{x^2+bx+c}$, where x^2+bx+c can not be factored further.

TRIGONOMETRIC IDENTITIES

RECIPROCAL IDENTITIES

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

PYTHAGOREAN IDENTITIES

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

SUM AND DIFFERENCE IDENTITIES

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y \pm \sin x \sin y$$

$$\tan(x + y) = \frac{\tan x \pm \tan y}{1 \pm \tan x \tan y}$$

DOUBLE-ANGLE IDENTITIES

$$\sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$$

HALF-ANGLE IDENTITIES

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

Final Value Theorem and Error

$$x(0) = \lim_{s \to \infty} sX(s)$$ Final Value Theorm (D.C Gain)

$$x(\infty) = \lim_{s \to 0} sX(s)$$ Initial Value Theorm

Static Error Constants

In the previous section we derived the following relationships for steady-state error.
For a step input, $u(t)$,

$$e(\infty) = e_{step}(\infty) = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

For a ramp input, $tu(t)$,

$$e(\infty) = e_{ramp}(\infty) = \frac{1}{\lim_{s \to 0} sG(s)}$$

For a parabolic input, $\frac{1}{2}t^2u(t)$,

$$e(\infty) = e_{parabola}(\infty) = \frac{1}{\lim_{s \to 0} s^2G(s)}$$

Where the type is the number of pure integrators in a system G(s)C(s)

Input	**Step**, A/s	**Ramp**, A/s^2	**Parabola**, A/s^3
Error Constant	$K_p = \lim_{s \to 0} G(s)$	$K_v = \lim_{s \to 0} sG(s)$	$K_a = \lim_{s \to 0} s^2G(s)$
System type		**Steady-State Error**	
0	$\frac{A}{1+K_p}$	∞	∞
1	0	$\frac{A}{K_v}$	∞
2	0	0	$\frac{A}{K_a}$
3 and higher	0	0	0

With Disturbance
https://www.ee.usyd.edu.au/tutorials_online/matlab/extras/ess/ess.html

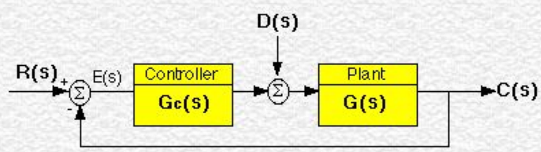
$$e(\infty) = \lim_{s\to 0} sE(s) = \lim_{s\to 0} \frac{s}{1+G_1(s)G_2(s)} R(s) - \lim_{s\to 0} \frac{sG_2(s)}{1+G_1(s)G_2(s)} D(s)$$
$$= e_R(\infty) + e_D(\infty)$$

where

$$e_R(\infty) = \lim_{s\to 0} \frac{s}{1+G_1(s)G_2(s)} R(s)$$

and

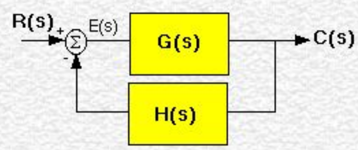
$$e_D(\infty) = -\lim_{s\to 0} \frac{sG_2(s)}{1+G_1(s)G_2(s)} D(s)$$



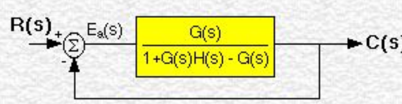
we can find the steady-state error for a step disturbance input with the following equation:

$$e(\infty) = \frac{1}{\lim_{s\to 0} \frac{1}{G(s)} + \lim_{s\to 0} G_s(s)}$$

Lastly, we can calculate steady-state error for non-unity feedback systems:



By manipulating the blocks, we can model the system as follows:

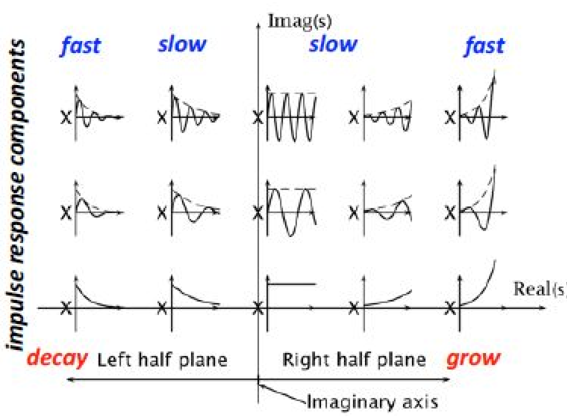


Now, simply apply the equations we talked about above.

Matching Output Responses with Time Domain Plot

In order

1. Stability (LHP Poles)
2. Final value
   - G(0) multiplied by whatever the time domain input is
3. Poles
   - Complex poles
     i. ω_D damped frequency is magnitude of imaginary part of pole
     ii. $\sigma \pm j\omega$ complex conjugate pole pair in the left half of the s-plane combines to generate a response component that is of the form $Ae^{-\sigma t}sin(\omega t + \Phi)$
     iii. $\frac{2\pi}{\omega_D}$ is the period of oscillation, match to plot
     iv. When the damped frequency ζ is small <0.1? The frequency of oscillation $\omega_d \approx \omega_0$
   - Check for
4. Zeroes
   - Undershoot when RHZ (Positive/Unstable)
   - Overshoot when LHZ (Negative/Stable) and small (close to imaginary axis) relative to dominant pole
5. Time Constant
   - Typically the dominant stable pole $(s + a)$ goes to e^{-at} where $\tau = \frac{1}{a}$
   - Expect almost full decay after 5 time constants



Bode Plots

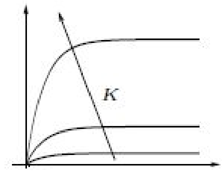
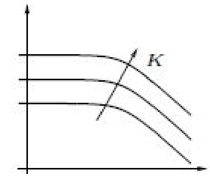
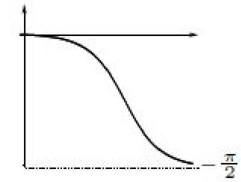
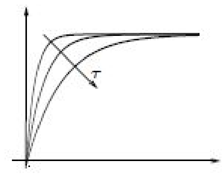
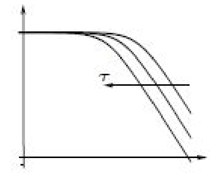
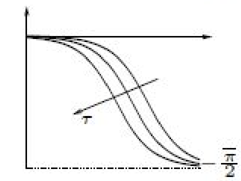
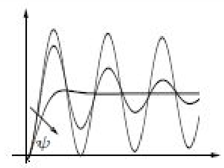
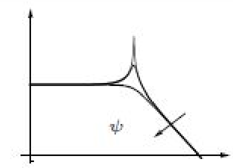
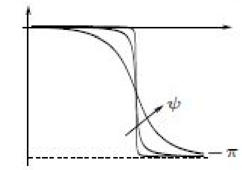
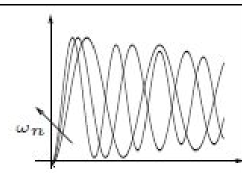
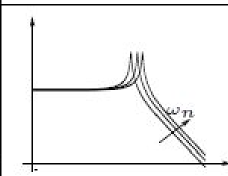
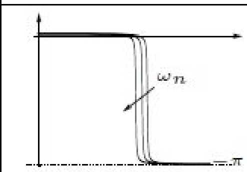
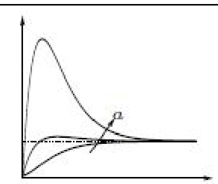
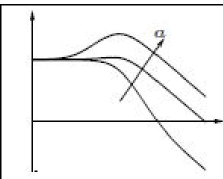
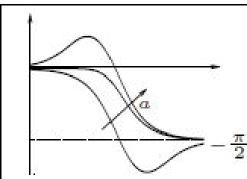
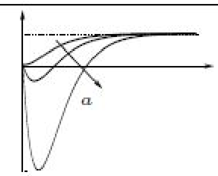
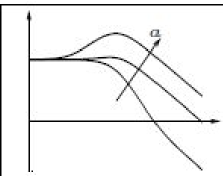
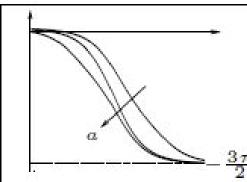
Checklist

1. Initial value $20log_{10}(|G(0)|$
2. Final magnitude rate of change is equal to $(n-m)*20dB/dec$
3. Total Phase Change
4. Existence and location of peak consistent with ω_0 when there are conjugate pair poles and $\zeta < 0.5$
5. Location of magnitude changes (usually a decade to either side of ω_0)
6. Location of phase changes (usually a decade of less to either side of ω_0)

Term	Magnitude	Phase
Constant: K	$20\log_{10}(\|K\|)$	K>0: 0° K<0: ±180°
Pole at Origin (Integrator) $\frac{1}{s}$	-20 dB/decade passing through 0 dB at ω=1	-90°
Zero at Origin (Differentiator) s	+20 dB/decade passing through 0 dB at ω=1 *(Mirror image of Integrator about 0 dB)*	+90° *(Mirror image of Integrator about 0°)*
Real Pole $\frac{1}{\frac{s}{\omega_0}+1}$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -20 dB/decade 3. Connect lines at ω_0.	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -90° 3. Connect with a straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$
Real Zero $\frac{s}{\omega_0}+1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +20 dB/decade 3. Connect lines at ω_0. *(Mirror image of Real Pole about 0 dB)*	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +90° 3. Connect with a straight line from $0.1\cdot\omega_0$ to $10\cdot\omega_0$ *(Mirror image of Real Pole about 0°)*

RHP Real Zero $(s-1)$	Same as above	1. Phase change of -90° in frequencies a decade either side of w=1 rad/sec
Underdamped Poles (Complex conjugate poles) $\frac{\omega_0^2}{s^2+2\zeta\omega_0 s+\omega_0^2}$ $\frac{1}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}$, $0<\zeta<1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at -40 dB/decade 3. If $\zeta<0.5$, then draw peak at ω_0 with amplitude $\lvert H(j\omega_0)\rvert=-20\cdot\log_{10}(2\zeta)$, else don't draw peak 4. Connect lines	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at -180° 3. Connect with straight line from $\omega=\frac{\omega_0}{10^\zeta}$ to $\omega_0\cdot 10^\zeta$
LHP Underdamped poles (Complex Conjugate Poles)	Same as above	1. Phase change of -180 around natural frequency ω_0 (half a decade to either side)
Underdamped Zeros (Complex conjugate zeros) $\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1$ $0<\zeta<1$	1. Draw low frequency asymptote at 0 dB 2. Draw high frequency asymptote at +40 dB/decade 3. If $\zeta<0.5$, then draw peak at ω_0 with amplitude $\lvert H(j\omega_0)\rvert=+20\cdot\log_{10}(2\zeta)$, else don't draw peak	1. Draw low frequency asymptote at 0° 2. Draw high frequency asymptote at +180° 3. Connect with straight line from $\omega=\frac{\omega_0}{10^\zeta}$ to $\omega_0\cdot 10^\zeta$

	4. Connect lines *(Mirror image of Underdamped Pole about 0 dB)*	*(Mirror image of Underdamped Pole about 0°)*

System	Parameter	Step response	Bode (gain)	Bode(phase)
$\frac{K}{\tau s + 1}$	K			
	τ			
$\frac{\omega_n^2}{s^2 + 2\psi\omega_n s + \omega^2}$	ψ			
	ω_n			
$\frac{as + 1}{(s+1)^2}$	a			
$\frac{-as + 1}{(s+1)^2}$	a			

Routh Criterion

If degree of characteristic polynomial is 1,2 then you can just check if there are any negatives. For higher order polynomials Routh Criterion can be used.

Special cases:

1. A zero in a row with at least one non-zero appearing later in the same row.
   - This means that there will be a sign change i.e an unstable pole
   - If you still want to know how many unstable poles, replace zero with epsilon and take the limit of epsilon going to zero, so it is still positive but very small.
2. Entire row is zeros - results in three possibilities
   - Two real roots equal and opposite in sign (Unstable)
   - Two imaginary roots that are complex conjugates (Marginally Stable)
   - Four roots that are all equal distance from the origin (Unstable)

For $p(s) = \sum_{k=0}^{4} a_k s^k,\ a_4 > 0,$

$$
\begin{array}{c|ccc}
s^4 & a_4 & a_2 & a_0 \\
s^3 & a_3 & a_1 & 0 \\
s^2 & b_0 \doteq -\frac{a_4 a_1 - a_2 a_3}{a_3} & b_1 \doteq -\frac{a_4 \cdot 0 - a_0 a_3}{a_3} = a_0 & 0 \\
s^1 & c \doteq -\frac{a_3 b_1 - a_1 b_0}{b_0} & 0 & \\
s^0 & -\frac{b_0 \cdot 0 - b_1 c}{c} = b_1 & &
\end{array}
$$

Root-Locus

1. There are n lines (loci) where n is the degree of Q or P (whichever is greater)
2. As K moves from 0->∞ the roots move from the poles of G(s) to the zeroes of G(s)
3. When roots are complex they occur in conjugate pairs
4. At no time will the same root cross over its own path
5. The portion to the left of an odd number of open loop poles/zeroes are part of the loci
6. Lines leave (breakout) and enter (breakin) the real axis at 90 degrees
7. If there are not enough poles or zeroes to make a pair then the extras go to or come from infinity
8. Lines go towards infinity along asymptotes dictated by the equation where
   Angle: $\Phi_A = \frac{(2q+1)}{n-m} * 180 \deg$

   Centroid: $\frac{\sum finite\ poles - \sum finite\ zeroes}{n-m}$

   $n - m = \#Poles\ -\ \#Zeroes$
9. If there are at least two lines to infinity, the sum of all roots is constant
10. K going from 0 to negative infinity can be drawn by reversing rule 5 and adding 180 degrees to the asymptote angles
11. Phase condition: the angle of a point on the root locus to all zeros minus the angle to all poles is equal to $(2l+1)\pi$

$$(2l+1)\pi = \angle F(s_0) = \sum_{k=1}^{m} \angle(s_0 - \beta_k) - \sum_{k=1}^{n} \angle(s_0 - \alpha_k) \text{ for } l = 0, \pm 1, \pm 2, \ldots$$

$$\angle(s - p_j) = 180° + \sum_{i=1}^{m} \angle(s - z_i) - \sum_{\substack{i=1 \\ i \neq j}}^{n} \angle(s - p_i)$$

Nyquist Plots

The system is only stable if:
Anticlockwise Encirclements of -1 = # Unstable RHP Poles

Plotting

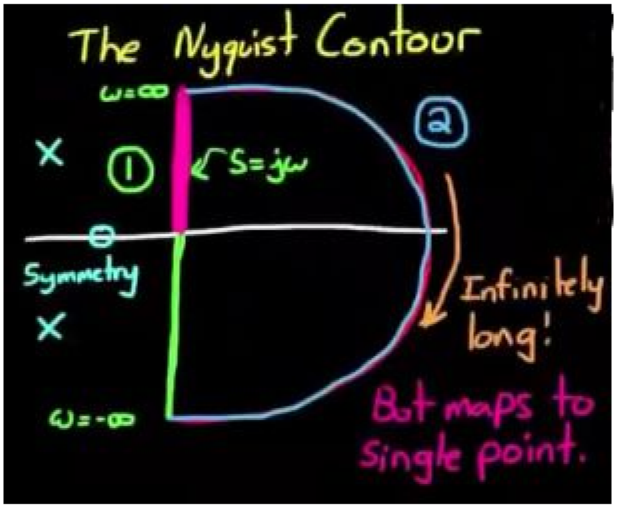
1. Put $s = j\omega$ into the transfer function
2. Sweep ω from 0 to ∞
3. Draw reflection about real axis

There are 4 points needed in order to do this

1. $\omega = 0$
   Plug 0 into the transfer function and evaluate, the Nyquist plot starts here
2. $\omega = \infty$
   Plug infinity into the transfer function and evaluate, the Nyquist plot ends here
3. Imaginary intercepts
   Plug $s = j\omega$ into the transfer function and split into the real and imaginary parts. Set the real part to zero and solve for ω, and then plug that into the imaginary part
4. Real intercepts
   Same as above, but set the imaginary part to zero and plug the frequency into the real part

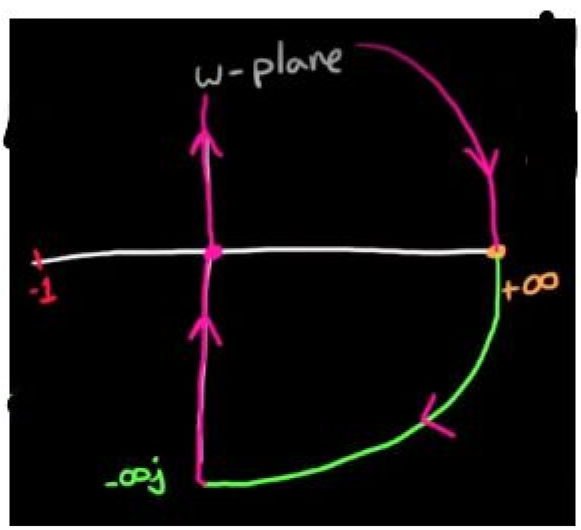
No Poles at the Origin

1. For a Proper transfer function, entire Nyquist contour for the infinity region maps to a single point with a finite magnitude on the positive real line.
2. For a strictly proper transfer function, entire Nyquist contour for the infinity region maps to zero



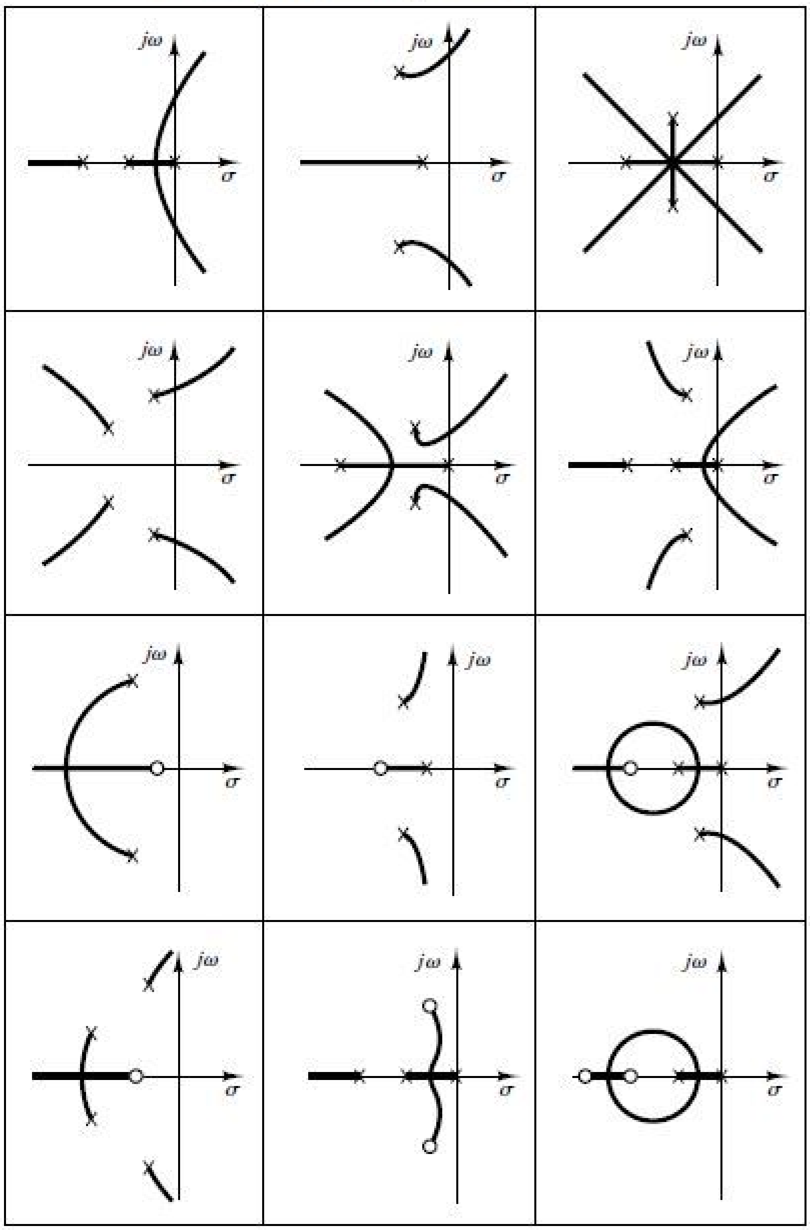
Poles at the Origin

1. Instead of going to the pole at the origin, encircle it with a very small radius ε
2. As this encirclement happens, the gain will tend to infinity
   a. If the encirclement is done on the right, it will exclude this pole and gain will be $+\infty$
   b. If the encirclement is done on the left, it will include the pole and gain will be $-\infty$
3. Phase will sweep from -90 to 90
4. The infinity part of the Nyquist contour should always be going clockwise?



Typical Pole–Zero Configurations and Corresponding Root Loci. In summarizing, we show several open-loop pole–zero configurations and their corresponding root loci in Table 6–1. The pattern of the root loci depends only on the relative separation of the open-loop poles and zeros. If the number of open-loop poles exceeds the number of finite zeros by three or more, there is a value of the gain K beyond which root loci enter the right-half s plane, and thus the system can become unstable. A stable system must have all its closed-loop poles in the left-half s plane.

Table 6–1 Open-Loop Pole–Zero Configurations and the Corresponding Root Loci



ECE 3510 Nyquist Plot Notes

A. Stolp
4/3/08,
4/9/09

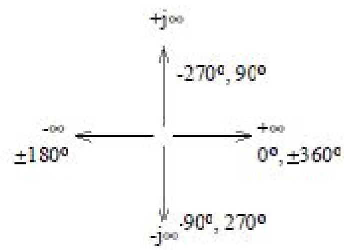
A Nyquist plot is essentially a polar Bode plot. Like a Bode plot, it is plotted for the Open-Loop (OL) Transfer function and will give information about the stability of the Closed-Loop (CL) system.

Open-Loop (OL) Transfer function: $G(s) = \frac{N_G(s)}{D_G(s)}$ m = number of zeros, n = number of poles

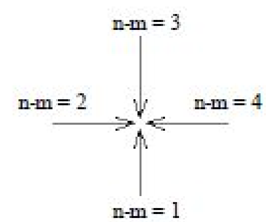
Basic Nyquist Rules

1. "Clean up" any "-s" terms in G(s) by multiplying by -1 as needed.
   If a "-" remains in G(s), the Nyquist plot will be mirrored about the imaginary axis. (rare)

2. Start at G(0), the DC gain, a point on the real axis.
   If G(s) has a zero at the origin: $G(0) = 0$
   If G(s) has a pole at the origin: $G(0) = \pm\infty$
   Check initial phase angle as you would for a Bode plot.



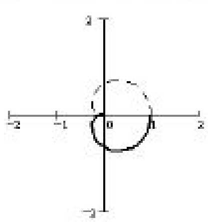
3. End at G(∞).
   - $n < m$ (rare) Plot ---> ∞ almost always +∞
   - $n = m$ Plot ---> G(∞), a point on the real axis
   - $n > m$ (most common) Plot ---> 0 Angle of approach to origin = $(n - m) \cdot (-90 \cdot deg)$



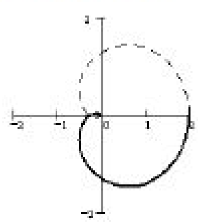
4. Plot the rest of the frequency response of G(s). It may help to start with Bode plots.

5. The $\omega < 0$ curve (dashed line) is simply the mirror image of the $\omega > 0$ curve about the real axis.
   This part of the curve is usually not necessary, it doesn't provide any more information.

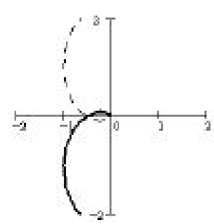
6. Gain, k, makes entire plot grow in all directions (or shrink if k<1).



--> bigger k -->



--> bigger k -->



7. $Z = N + P$
   P = OL poles in RHP (0 if open-loop stable)
   N = CW encirclements of -1, CCW encirclements are counted as negative and may make up for P.
   Z = CL poles in RHP (must be zero (or ≤ 0) if closed-loop stable)

8. ANY CW encirclements means Closed-Loop system is UNSTABLE
   $N > 0$ --> CL unstable

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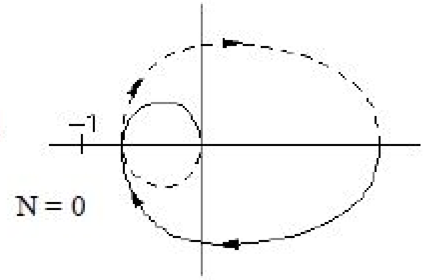
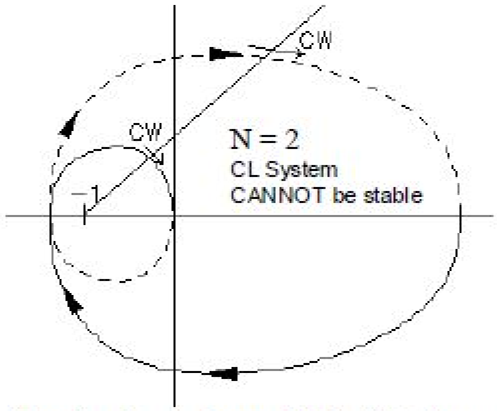
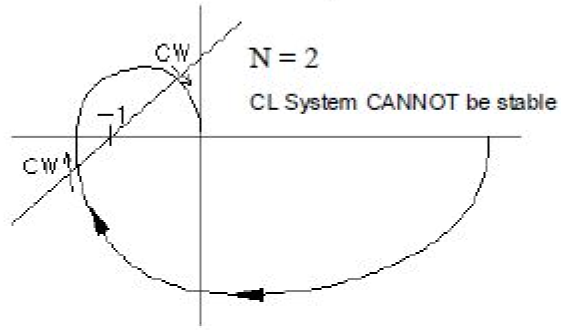
Counting Clockwise Encirclements

N = CW encirclements of -1,
CCW encirclements are counted as negative and may make up for P.

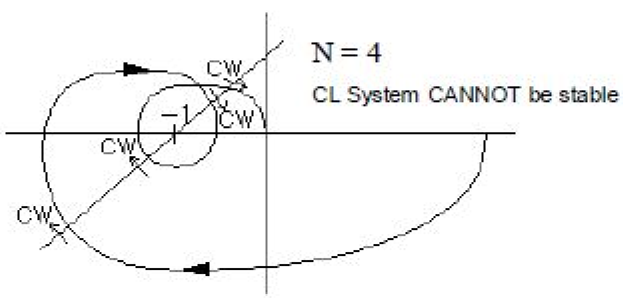


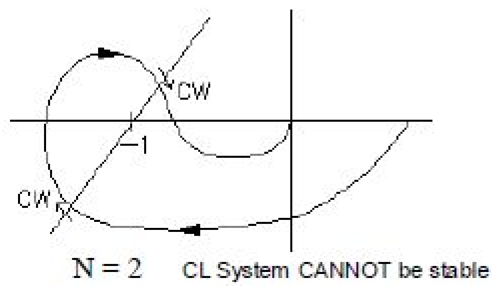


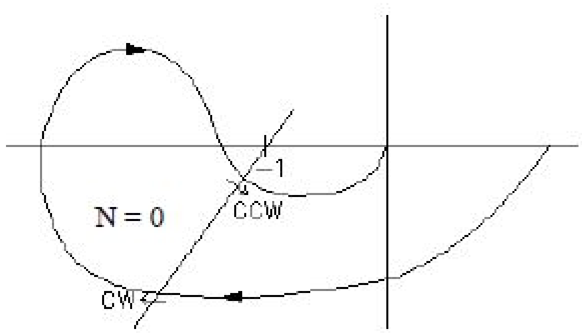


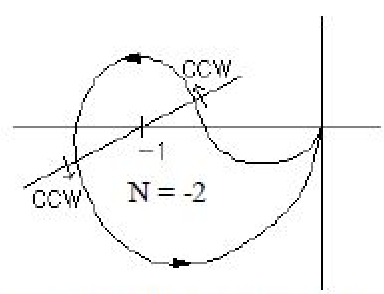
If you have the ω < 0 curve (dashed line), then you can use any single-ended line that starts at -1 to help you count encirclements.

If you don't have the ω < 0 curve (dashed line), then make your line extend both directions from -1.









CCW encirclements are counted as negative.

CL System CAN be stable, if $P \leq 2$

$-N$ can make up for $+P$. and stabilize an OL unstable system

$$Z = N + P$$

P = OL poles in RHP (0 if open-loop stable)

N = CW encirclements of -1. CL System CANNOT be stable if $N > 0$

Z = CL poles in RHP (must be zero (or ≤ 0) if closed-loop stable)

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Gain Margin (GM) and Phase Margin (PM)

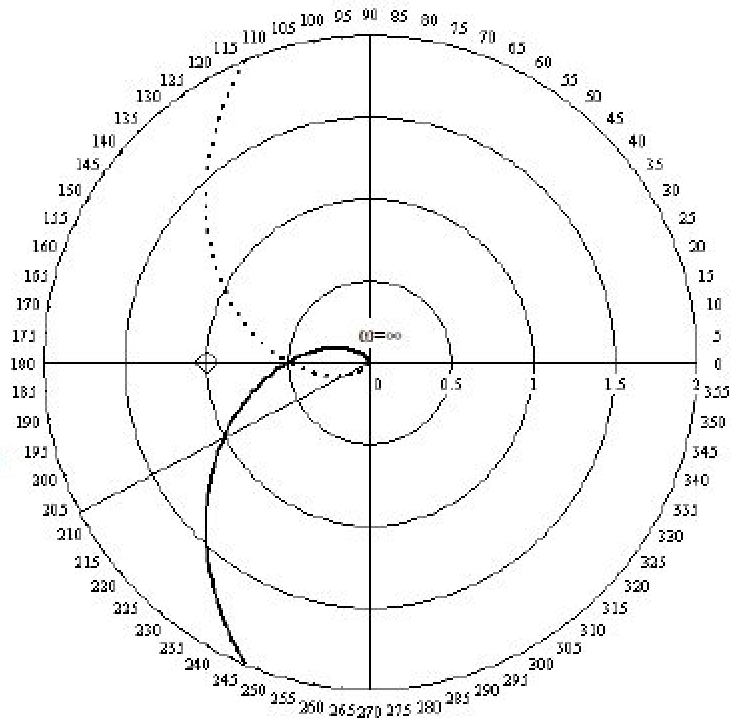
ECE 3510 Nyquist Plot Notes p.3

To find the Phase Margin (PM):

1. Find where the Nyquist plot crosses the unit circle. These crossings separate the unit circle into regions.
2. Decide which of these regions have unacceptable CW encirclements.
3. Determine what phase change would cause the -1 point to be an unacceptable region, usually 180^0 - <u>/ crossing</u>

$$GM = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$PM = 207 - 180 = 27 \text{ deg}$$



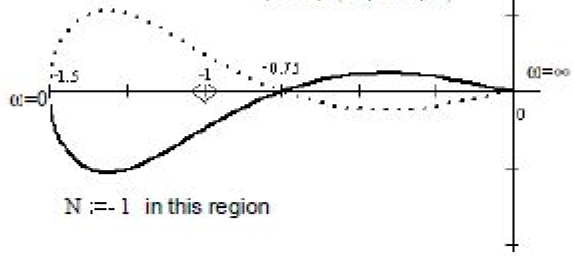
To find the Gain Margin (GM):

1. Find where the Nyquist plot crosses the negative real axis. These crossings separate the negative real axis into regions.
2. Decide which of these regions have unacceptable CW encirclements.
3. Determine what gain would cause the -1 point to be an unacceptable region, usually $\frac{1}{\text{- crossing}}$ into the unacceptable region.
4. Usually there is just one upper limit of gain– in that case report that as the Gain Margin.
5. If there is a lower limit of gain, report the Gain Margin as: $GM = \left[\text{Lower limit , upper limit} \right]$

If there is no upper limit, then report it as ∞

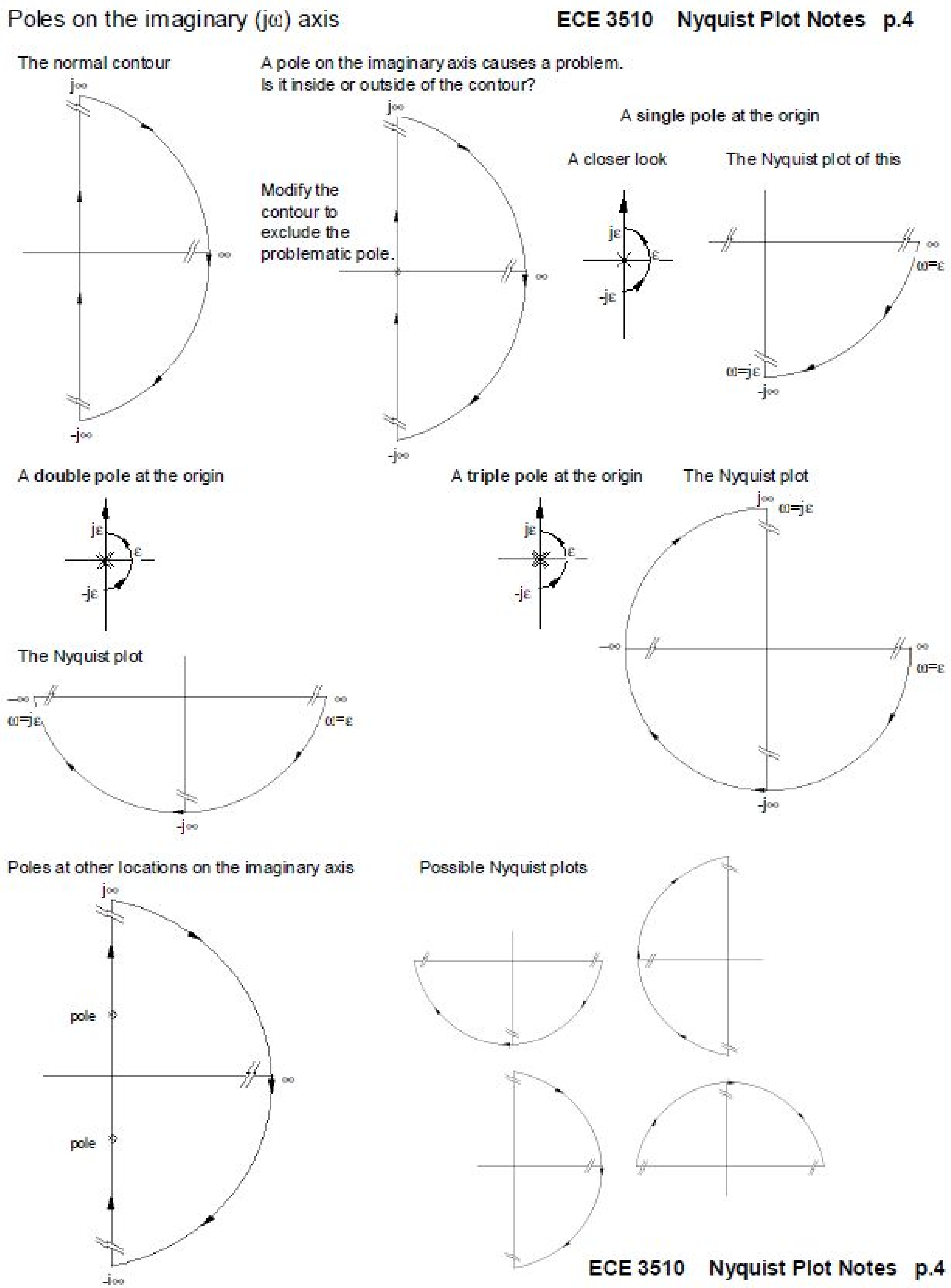
$$G(s) := \frac{1.5\cdot(s+2)}{(s-1)\cdot\left(s^2+2\cdot s+2\right)}$$

$P := 1$ For CL stability, $N := -1$ or more



$N := -1$ in this region

$$GM = \left[\frac{1}{1.5} , \frac{1}{0.75} \right] = \left[0.667 , 1.333 \right]$$


Poles on the imaginary (jω) axis
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The normal contour
j∞
∞
-j∞
A pole on the imaginary axis causes a problem.
Is it inside or outside of the contour?
Modify the contour to exclude the problematic pole.
A single pole at the origin
A closer look
jε
ε
-jε
The Nyquist plot of this
ω=ε
ω=jε
-j∞
A double pole at the origin
The Nyquist plot
−∞
ω=jε
ω=ε
A triple pole at the origin
The Nyquist plot
ω=jε
Poles at other locations on the imaginary axis
pole
pole
Possible Nyquist plots
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